

Adaptive Control Design Methodology for Nonlinear-in-Control Systems in Aircraft Applications

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An adaptive control design method for both short-period and lateral/directional control of a fighter aircraft is presented. The proposed method is directly applicable to flight control of vehicles with state-dependent and nonlinear-in-control uncertain dynamics. The developed design approach uses a specialized set of radial basis function approximators and Lyapunov-based adaptive laws to compensate for the unknown control-dependent nonlinearities. The adaptive controller is defined as a solution of fast dynamics, which verifies the assumptions of Tikhonov's theorem from singular perturbations theory. Simulations illustrate the theoretical findings.

I. Introduction

TRADITIONAL flight control design relies on linearization of the equations of motion around a set of trim points (equilibrium), which are subsequently used to design gain-scheduled controllers that achieve desired performance specifications throughout the vehicle flight envelope. The linear approximations to the aircraft dynamics are often derived under the assumption that the short-period dynamics is faster than the aircraft phugoid mode. To substantiate a gain-scheduling design for the roll/yaw axes, it is customary to assume that the corresponding roll/yaw modes are faster than the aircraft spiral mode. Often during the control design phase, the aircraft roll dynamics is assumed to be faster than and independent of the Dutch-roll motion [1]. For high angles of attack and other aggressive flight regimes in which fighter aircraft often operate, these assumptions may no longer hold and the corresponding linear approximations are seen to deviate from flight test data (see, for example, [1]). Moreover, the dynamic nonlinearities in these aggressive flight regimes are dependent not only on the states of the system, but also on the control inputs [2]. Additionally, it is well known that modern fighter aircraft may experience degradation in aerodynamic stability and control with increased angle of attack α , which can lead to dangerous departure/spin conditions and loss of control. Some high- α conditions include wing rock, roll reversal due to low directional stability and adverse yaw, control-induced departures, and directional divergence (see, for instance, [3]).

The design of control systems for flight regimes involving significant control-dependent nonlinearities is relatively less developed [3]. One such method is introduced in [4] based on nonlinear model predictive control, which expands the output and control vectors into a truncated Taylor series and further applies

optimal control design. The control law uses the inverse of a matrix that depends on the prediction horizon, relative degree of the system, and the order of the Taylor series expansion of the output. The tradeoff of the method is between the number of terms in the Taylor series expansion toward improvement of the modeling accuracy and the selection of the appropriate prediction horizon to achieve a suitable control penalty. An alternative control method to gain scheduling is feedback linearization via dynamic inversion, which holds the potential for addressing the system nonlinearities and for reducing control development costs [5]. Nonlinear control methods for linear-in-control systems using dynamic inversion can be found in [6,7]. These authors have used the natural time-scale separation of the aircraft's dynamics to design robust controllers for a certain class of uncertainties.

Flight tests with neural network based adaptive controllers have shown great promise for compensation of component failures and modeling uncertainties [8,9]. Neural network approximation based adaptive controllers for systems with control-dependent nonlinearities are introduced in [5,10,11]. For these types of systems, one of the design challenges is the necessity to resolve (online) an implicit algebraic loop that arises in the formulation of the corresponding adaptive controller. Kim and Calise [5] solve this algebraic implicit control loop at every time step using the previous control value. In [11], the authors differentiate the system dynamics with respect to time so that the derivative of the control input appears linearly and further apply linear control design methods. Then the actual control signal is computed via integration. Although the controller in [11] is shown to exhibit good tracking performance, it assumes that the nonlinearities are globally linearly parameterized in unknown parameters via known basis functions. If the nonlinearities are unstructured, then such a parameterization becomes invalid, and one has to consider local approximation, for which the approximation errors become dependent on the control input and the system states. In that case, the method of [11] cannot be straightforwardly applied. A similar attempt has been made in [10].

In this paper, a direct adaptive model reference control design is proposed for multi-input/multi-output (MIMO) systems, whose dynamics may contain both state and control-dependent uncertainties. This nonlinear-in-control adaptive design was originally developed in [12]. The main contribution of this paper is in the application of the nonlinear-in-control design methodology to short-period and roll/yaw dynamics of a fighter-type aircraft. A specialized set of radial basis functions is introduced to approximate the control-dependent nonlinearities on a local bounded domain of interest of initial conditions. The main feature of this new class of approximators is that they retain the same sign of the control

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effectiveness of the original system dynamics with appropriately designed adaptive laws.

The rest of the paper is organized as follows. In Sec. II, we apply the theory developed in [12] to a nonlinear-in-control fighter aircraft such as the F-16 flying at or near stall angle of attack in the presence of aerodynamic uncertainties. In Sec. III, we further extend the method developed in [13] to uncertain nonlinear-in-control MIMO systems, which describe the F-16 lateral/directional dynamics at different angles of sideslip and in the presence of nonlinear uncertainties. In both systems, the algebraic loop for the definition of the adaptive increment inherent to the method in [5] is completely avoided. Simulations illustrate the theoretical results. Moreover, the proposed design is shown to be applicable to a more general class of nonlinearities as compared to [11]. A brief summary concludes the paper in Sec. IV.

II. Adaptive Control Design for Nonlinear-In-Control Single-Input Aircraft System

A. Aircraft Short-Period Dynamics

Neglecting the influence of gravity and thrust, the short-period dynamics of a rigid aircraft can be described as

$$\begin{aligned}\dot{\alpha} &= -\frac{L_\alpha(\alpha)}{V}\alpha + q - \frac{L_{\delta_e}(\alpha, \delta_e)}{V}\delta_e \\ \dot{q} &= M_0(\alpha) + M_q(\alpha_0)q + M_{\delta_e}(\alpha, \delta_e)\delta_e\end{aligned}\quad (1)$$

where α is the angle of attack (AOA), q is the pitch rate, α_0 is a trimmed AOA, δ_e is the elevator deflection, $L_\alpha(\alpha)$ is the known lift curve slope at α , $L_{\delta_e}(\alpha, \delta_e)$ is the known lift effectiveness due to elevator deflection, and V is the trimmed airspeed. Without loss of generality and to simplify further discussions, the aircraft pitch damping term $M_q(\alpha_0)$ is assumed to be known. Also in Eq. (1), $M_0(\alpha)$ is the aircraft pitching moment at $\delta_e = 0$, and $M_{\delta_e}(\alpha, \delta_e)$ represents the aircraft elevator effectiveness, which depends nonlinearly on AOA and δ_e [14]. Often in practice, the pitching moment components $M_0(\alpha)$ and $M_{\delta_e}(\alpha, \delta_e)$ are generally unknown. However, some partial knowledge of the aerodynamic stability and control derivatives is usually available from wind-tunnel experiments and/or theoretical predictions. Incorporating prior known data, the short-period dynamics of Eq. (1) can be rewritten as

$$\begin{aligned}\dot{\alpha} &= -\frac{L_\alpha(\alpha)}{V}\alpha + q - \frac{L_{\delta_e}(\alpha, \delta_e)}{V}\delta_e \\ \dot{q} &= M_\alpha(\alpha_0)\alpha + M_q(\alpha_0)q + M_{\delta_e}(\alpha_0, 0)\delta_e + \underbrace{(M_0(\alpha) - M_\alpha(\alpha_0)\alpha)}_{M_{\delta_e} \Delta M_0(\alpha)} \\ &\quad + \underbrace{(M_{\delta_e}(\alpha, \delta_e) - M_{\delta_e}(\alpha_0, 0), \delta_e)}_{M_{\delta_e} \Delta M_{\delta_e}(\alpha, \delta_e)}\end{aligned}\quad (2)$$

where, with a slight abuse of notation, M_{δ_e} denotes the constant

$$M_{\delta_e} = \left. \frac{\partial M_{\delta_e}(\alpha, \delta_e)}{\partial \delta_e} \right|_{\alpha=\alpha_0, \delta_e=0}$$

that results from linearization and it is assumed that $M_\alpha(\alpha_0)$ and $M_{\delta_e}(\alpha_0, 0)$ are the known constant stability and control derivatives. The lift derivative $L_{\delta_e}(\alpha)$ is assumed to be small with respect to the true airspeed V , so that for control design purposes,

$$\frac{L_{\delta_e}(\alpha, \delta_e)}{V} \approx 0$$

which leads to the following model formulation:

$$\begin{cases} \dot{\alpha} = -\frac{L_\alpha}{V}\alpha + q \\ \dot{q} = M_\alpha(\alpha_0) + M_q(\alpha_0)q + M_{\delta_e}(\alpha_0)\delta_e + M_{\delta_e}(\alpha_0)(\underbrace{\Delta M_0(\alpha) + \Delta M_{\delta_e}(\alpha, \delta_e)}_{f(\alpha, \delta_e)}) \end{cases}$$

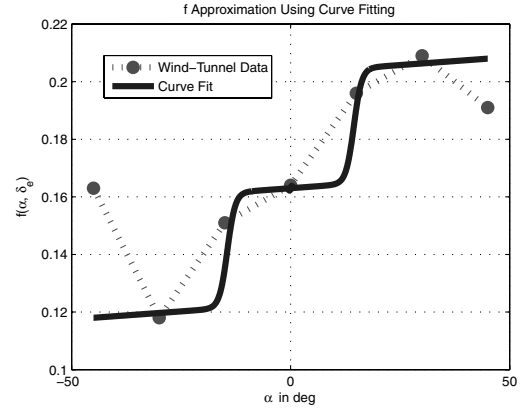


Fig. 1 $f(\alpha, \delta_e)$ as a function of α .

This can be rewritten in state space form:

$$\begin{aligned}\begin{bmatrix} \dot{\alpha}(t) \\ \dot{q}(t) \end{bmatrix} &= \underbrace{\begin{bmatrix} -\frac{L_\alpha}{V} & 1 \\ M_\alpha & M_q \end{bmatrix}}_{A_p} \begin{bmatrix} \alpha(t) \\ q(t) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ M_{\delta_e} \end{bmatrix}}_{b_p} (\delta_e(t) + f(\alpha(t), \delta_e(t))) \\ y(t) &= \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{c_p^T} \begin{bmatrix} \alpha(t) \\ q(t) \end{bmatrix}\end{aligned}\quad (3)$$

where $f(\alpha, \delta_e)$ represents the system matched and unknown nonlinear-in-control effects. The control *objective* is to design the elevator

$$\delta_e(t) = \delta_{e_{nom}}(t) - \delta_{e_{ad}}(t) \quad (4)$$

so that the angle of attack $\alpha(t)$ tracks a bounded, time-varying command $y^{cmd}(t)$ in the presence of the system uncertainties and control nonlinearities while all other signals in the system remain bounded. In Eq. (4), $\delta_{e_{nom}}(t)$ is the nominal control elevator deflection for the nominal (known) linear model, while $\delta_{e_{ad}}(t)$ is the incremental adaptive augmentation which will be designed to compensate for the unknown $f(\alpha(t), \delta_e(t))$.

Using wind-tunnel data from NASA Langley Research Center's wind-tunnel tests found in [14] on a subscale model of an F-16 airplane for flight regimes up to angle of attack of 45 deg and curve-fitting methods, the unknown function $f(\alpha, \delta_e)$ for the short-period dynamics can be approximated by

$$\begin{aligned}f(\alpha, \delta_e) &= ((1 - C_0)e^{-\frac{(\alpha-\alpha_0)^2}{2\sigma^2}} + C_0)(\tanh(\delta_e + h) \\ &\quad + \tanh(\delta_e - h) + 0.01\delta_e)\end{aligned}\quad (5)$$

which is well defined for all $\alpha, \delta_e \in \mathbb{R}$ (see Fig. 1). Here, $e^{-(\alpha-\alpha_0)^2/2\sigma^2}$ is the Gaussian function with the width σ , and C_0 and h are positive constants. If the angle of attack is at the trim angle, then full elevator control effectiveness is obtained. If the control is small, the control surface is within the boundary layer surface and does not produce an aerodynamic moment, portrayed in the small elevator control-induced moment term $0.01\delta_e$. As the control effort gets large, the surface stalls, the system reaches saturation and stops producing any additional moment, which is approximated by the hyperbolic tangent functions.

B. Ideal Reference Model

To find the nominal controller for the short-period aircraft dynamics (1), we consider the nominal linear model of the longitudinal dynamics (3) in the absence of uncertainties [$f(\alpha(t), \delta_e(t)) = 0, \delta_{e_{ad}} = 0$] and design a proportional + integral (PI) linear quadratic regulator (LQR) controller to achieve the AOA tracking objective: $\alpha(t) \rightarrow y^{\text{cmd}}(t)$ as $t \rightarrow \infty$ for a bounded and possibly time-varying AOA command $y^{\text{cmd}}(t)$. Toward that end, let α_I denote the integrated AOA tracking error defined as

$$\dot{\alpha}_I(t) = \alpha(t) - y^{\text{cmd}}(t)$$

Augmented with this additional state, the nominal system dynamics are defined as

$$\begin{aligned} \begin{bmatrix} \dot{\alpha}_I(t) \\ \dot{\alpha}_r(t) \\ \dot{q}_r(t) \end{bmatrix} &= \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{L_\alpha}{V} & 1 \\ 0 & M_\alpha & M_q \end{bmatrix}}_A \begin{bmatrix} \alpha_I(t) \\ \alpha_r(t) \\ q_r(t) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ M_{\delta_e} \end{bmatrix}}_b \delta_{e_{\text{nom}}}(t) \\ &+ \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} y^{\text{cmd}}(t) \end{aligned} \quad (6)$$

Let

$$A_r = A - b k_{\text{LQR}} \quad (7)$$

where the gain $k_{\text{LQR}} = R^{-1} b^T P$ is found by solving the corresponding Riccati equation for the unique positive definite symmetric matrix $P = P^T > 0$:

$$Q + P A + A^T P - P b R^{-1} b^T P = 0$$

in which $Q \geq 0$ is a positive semidefinite matrix and $R > 0$ is a scalar (control weight). This leads to the following form of the closed-loop reference system for the short-period aircraft dynamics (3):

$$\begin{bmatrix} \dot{\alpha}_I(t) \\ \dot{\alpha}_r(t) \\ \dot{q}_r(t) \end{bmatrix} = A_r \begin{bmatrix} \alpha_I(t) \\ \alpha_r(t) \\ q_r(t) \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} y^{\text{cmd}}(t) \quad (8)$$

The reference model dynamics (8) is chosen to represent the nominal system (6) operating under the baseline LQR PI controller. It follows then that the nominal control command for the elevator angle is

$$\delta_{e_{\text{nom}}}(t) = -k_{\text{LQR}}^T \begin{bmatrix} \alpha_I(t) \\ \alpha_r(t) \\ q_r(t) \end{bmatrix} \quad (9)$$

which can be applied to the dynamics in Eq. (3) to yield

$$\begin{aligned} \begin{bmatrix} \dot{\alpha}_I(t) \\ \dot{\alpha}(t) \\ \dot{q}(t) \end{bmatrix} &= A_r \begin{bmatrix} \alpha_I(t) \\ \alpha(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} y^{\text{cmd}}(t) + b(-\delta_{e_{ad}}(t)) \\ &+ f(\alpha(t), \delta_{e_{\text{nom}}}(t) - \delta_{e_{ad}}(t)) \\ \alpha_I(0) &= \alpha_{I_0}, \quad \alpha(0) = \alpha_0, \quad q(0) = q_0 \end{aligned} \quad (10)$$

C. RBF Approximation

In this section, we apply the methodology of [12] for the approximation of monotonic functions which uses the main result from [15]. Let

$$\begin{aligned} g(\alpha, \delta_e) &= \frac{\partial f}{\partial \delta_e}(\alpha, \delta_e) \\ &= ((1 - C_0) e^{\frac{(\alpha - \alpha_0)^2}{2\sigma^2}} + C_0)(2.01 - \tanh^2(\delta_e + h) - \tanh^2(\delta_e - h)) \\ &\geq 0.01 C_0 > 0 \quad \text{for all } C_0 > 0 \end{aligned} \quad (11)$$

Using the Leibnitz formula, $f(\alpha, \delta_e)$ can be expressed as

$$f(\alpha, \delta_e) = f(\alpha, 0) + \int_0^{\delta_e} g(\alpha, \xi) d\xi \quad (12)$$

where the integral term $g(\alpha, \xi) > 0$ is bounded away from zero for all values of α and ξ as seen in Eq. (11). The main theorem of [15] implies that each component of $f(\alpha, \delta_e)$, defined in Eq. (12), can be approximated by a linear combination of radial basis functions (RBFs) arbitrarily close on a compact set. Moreover, the constructive proof in [15] implies that the approximation of $g(\alpha, \delta_e) > 0$ will be achieved via positive coefficients. Thus, consider a set of RBFs $\phi: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ given by

$$\phi(\alpha, \delta_e) = \phi_1(\alpha) + \phi_2(\alpha, \delta_e) \quad (13)$$

where

$$\phi_1(\alpha) \triangleq \theta^T \Phi_1(\alpha) = \sum_{i=1}^N \theta_i e^{-\frac{(\alpha - \alpha_{c_i})^2}{\delta_i^2}}, \quad \delta_i > 0 \quad (14)$$

$$\begin{aligned} \phi_2(\alpha, \delta_e) &\triangleq w^T \Phi_2(\alpha, \delta_e) = \sum_{j=1}^M w_j \left(\int_0^{\delta_e} e^{-\frac{(\chi - \chi_{c_j})^2}{\rho_j^2}} d\chi \right) \\ \chi &\triangleq [\alpha^T \ \xi]^T, \quad \rho_j > 0 \end{aligned} \quad (15)$$

and M and N are positive constants, $\phi_1(\alpha)$ is the vector of Gaussians dependent only on α , and $\phi_2(\alpha, \delta_e)$ is the vector of the integrals of the Gaussians dependent on both α and δ_e . Because $g(\alpha, \delta_e) > 0$, then following [15], the unknown coefficients w_i are positive. The vectors $\alpha_{c_i}, i = 1, \dots, N, \chi_{c_j} = [\alpha_{c_j}^T \ \delta_{e_{c_j}}]^T, j = 1, \dots, M$ represent the fixed centers of the RBFs, δ_i and ρ_j are the fixed widths of the basis functions, respectively, and the unknown parameters are $\theta_i \in \mathbb{R}$ and $w_j \in \mathbb{R}^+$. It is straightforward to see from Eqs. (13–15) that

$$\text{sgn} \left(\frac{\partial \phi}{\partial \delta_e}(\alpha, \delta_e) \right) = \text{sgn} \left(\frac{\partial \phi_2}{\partial \delta_e}(\alpha, \delta_e) \right) > 0 \quad (16)$$

Then,

$$f(\alpha, \delta_e) = f(\alpha, 0) + \int_0^{\delta_e} g(\alpha, \xi) d\xi = W^T \Phi(\alpha, \delta_e) + \varepsilon(\alpha, \delta_e)$$

for $(\alpha, \delta_e) \in \Omega_\alpha \times \Omega_{\delta_e}$ and $|\varepsilon(\alpha, \delta_e)| \leq \varepsilon^*$, where $g(\alpha, \delta_e)$ is given in Eq. (11), $W^T = [\theta^T \ w^T]^T$, $\Phi(\alpha, \delta_e) = [\Phi_1^T(\alpha) \ \Phi_2^T(\alpha, \delta_e)]^T$, $w_i > 0$, $\Omega_\alpha, \Omega_{\delta_e}$ are compact sets of \mathbb{R} , and ε^* is a constant.

D. State Predictor and Adaptive Law

A state predictor for the system (10) can be constructed as

$$\begin{aligned} \begin{bmatrix} \dot{\hat{\alpha}}_I(t) \\ \dot{\hat{\alpha}}(t) \\ \dot{\hat{q}}(t) \end{bmatrix} &= A_r \begin{bmatrix} \hat{\alpha}_I(t) \\ \hat{\alpha}(t) \\ \hat{q}(t) \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} y^{\text{cmd}}(t) + b(-\delta_{e_{ad}}(t)) \\ &+ \hat{W}^T(t) \Phi(\alpha(t), \delta_e(t)) \\ \hat{\alpha}_I(0) &= \hat{\alpha}_{I_0}, \quad \hat{\alpha}(0) = \hat{\alpha}_0, \quad \hat{q}(0) = \hat{q}_0 \end{aligned} \quad (17)$$

where A_r is defined in Eq. (7), b is given in Eq. (6), and $\hat{W}(t)$ is the estimate of the unknown constant vector W . The prediction error signal $e_s(t)$ is defined as

$$e_s(t) = \begin{bmatrix} \hat{\alpha}_I(t) - \alpha_I(t) \\ \hat{\alpha}(t) - \alpha(t) \\ \hat{q}(t) - q(t) \end{bmatrix} \quad (18)$$

and the prediction error dynamics can be written as

$$\begin{aligned} \dot{e}_s(t) &= A_r e_s(t) + b(\tilde{W}^\top(t) \Phi(\alpha(t), \delta_e(t)) - \varepsilon(\alpha(t), \delta_e(t))) \\ e_{s,1}(0) &= \hat{\alpha}_{I_0} - \alpha_{I_0}, \quad e_{s,2}(0) = \hat{\alpha}_0 - \alpha_0, \quad e_{s,3}(0) = \hat{q}_0 - q_0 \end{aligned} \quad (19)$$

where $\tilde{W}(t) = \hat{W}(t) - W$. The adaptive law

$$\dot{\hat{W}}(t) = \Gamma \text{Proj}(\hat{W}(t), -\Phi(\alpha(t), \delta_e(t)) e_s^\top(t) P_0 B), \quad \hat{W}(0) = W_0 \quad (20)$$

where Γ is a positive definite matrix of adaptation rates, $\text{Proj}(\cdot, \cdot)$ denotes the projection operator, and P_0 is the solution to the Lyapunov equation $A_r^\top P_0 + P_0 A_r = -Q_0$ for a positive definite matrix Q_0 ensures that the parameter estimation error $\tilde{W}(t)$ is ultimately bounded [16].

Remark 1: Because the ideal weights w_i are positive, the compact set in the application of the projection operator can be selected to ensure that $\hat{w}_i(t)$ remain positive for all $t \geq 0$; that is, $\hat{w}_i(t) > \hat{w}_{i_0} > 0$ for all $i = 1, \dots, M$.

Remark 2: Notice that boundedness of the parameter error $\tilde{W}(t)$ does not imply stability of the overall system. It must be proven additionally that the state predictor dynamics (17) remain bounded in the presence of feedback.

E. Nonlinear Control Design

Define the tracking error between the predictor of Eq. (17) and the reference system (8) as

$$e(t) = \begin{bmatrix} \hat{\alpha}_I(t) - \alpha_{I_r}(t) \\ \hat{\alpha}(t) - \alpha_r(t) \\ \hat{q}(t) - q_r(t) \end{bmatrix} \quad (21)$$

The error dynamics are

$$\dot{e}(t) = A_r e(t) + b(-\delta_{e_{ad}}(t) + \hat{W}^\top(t) \Phi(\alpha(t), \delta_e(t))) \quad (22)$$

Our intent is to define the adaptive augmentation $\delta_{e_{ad}}$ such that the error dynamics in Eq. (22) become uniformly ultimately bounded. If the ideal adaptive controller is defined as the solution of the following implicit algebraic equation

$$\delta_{e_{ad}}(t) = \hat{W}^\top(t) \Phi(\alpha(t), \delta_{e_{nom}}(t) - \delta_{e_{ad}}(t)) \quad (23)$$

then the resulting closed-loop error dynamics are asymptotically stable, that is, $\dot{e}(t) = A_r e(t)$. Because of the nonlinear-in-control nature of the relation, Eq. (23) cannot be solved explicitly for $\delta_{e_{ad}}$. Instead, we construct fast dynamics to approximate the ideal solution for $\delta_{e_{ad}}$:

$$\epsilon \dot{\delta}_{e_{ad}} = -\text{sign}\left(\frac{\partial \mathbf{f}}{\partial \delta_{e_{ad}}}\right) \mathbf{f}(t, e, \delta_{e_{ad}}) \quad (24)$$

where $0 < \epsilon \ll 1$ and

$$\mathbf{f}(t, e, \delta_{e_{ad}}) = \delta_{e_{ad}}(t) - \hat{W}^\top(t) \Phi(\alpha(t), \delta_{e_{nom}}(t) - \delta_{e_{ad}}(t))$$

Figure 2 illustrates the approach.

Let $\delta_{e_{ad}}(t) = h(t, e)$ be an isolated root of $\mathbf{f}(t, e, \delta_{e_{ad}}) = 0$ and let $v(t, \alpha) = \delta_{e_{ad}}(t) - h(t, \alpha)$. The existence of such a root is guaranteed by noting that there must exist a point of intersection between $\hat{W}^\top(t) \Phi(\alpha(t), \delta_{e_{nom}}(t) - \delta_{e_{ad}}(t))$, and $\delta_{e_{ad}}(t)$ for every value of $\hat{W}(t)$. An illustration of this is shown in Fig. 3, which sketches $\hat{W}^\top(t) \Phi(\alpha(t), \delta_{e_{nom}}(t) - \delta_{e_{ad}}(t))$ and $\delta_{e_{ad}}$ as functions of $\delta_{e_{ad}}$ for one particular $\Phi(\alpha, \delta_{e_{nom}} - \delta_{e_{ad}})$. We note that the dynamics in Eqs. (22) and (24)

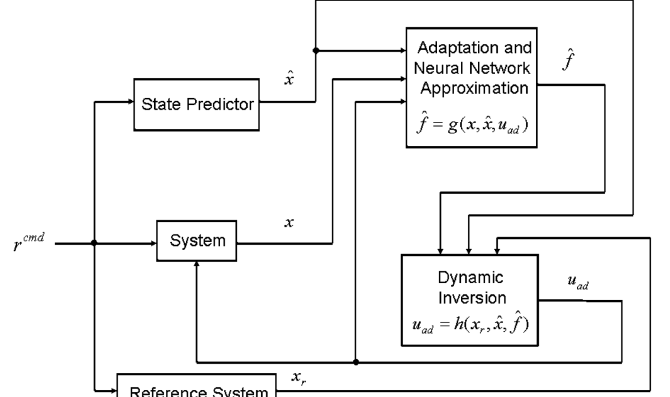


Fig. 2 Adaptive control design for nonlinear-in-control systems.

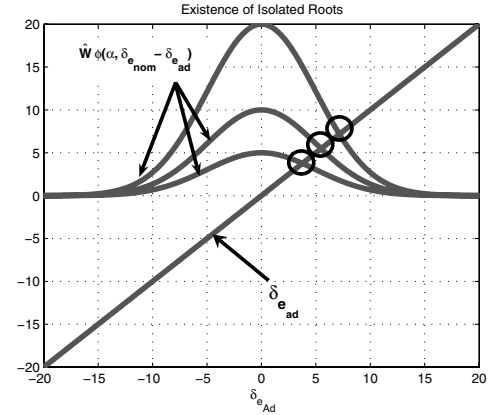


Fig. 3 Intersection of the functions $\delta_{e_{ad}}$ and $\hat{W}^\top \Phi(\alpha, \delta_{e_{nom}} - \delta_{e_{ad}})$.

present a singularly perturbed system, the stability analysis of which can be addressed using Tikhonov's theorem from singular perturbations theory [17]. Following [17], the reduced system is

$$\dot{e}(t) = A_r e(t), \quad e(0) = e_0$$

where $e(0) = [e_1(0) \ e_2(0) \ e_3(0)]^\top$, and the *boundary layer* system is

$$\frac{dv}{d\tau} = -\text{sign}\left(\frac{\partial \mathbf{f}}{\partial \delta_{e_{ad}}}\right) \mathbf{f}(t, e, v + h(t, e)) \quad (25)$$

where $\tau = t/\epsilon$. Following Remark 1, linearization of Eq. (25) with respect to v around the origin implies that the boundary layer system has locally exponentially stable origin, so that Tikhonov's theorem from singular perturbations theory can be applied [17]. Thus, the complete controller consists of Eqs. (17), (20), (22), and (24), and Tikhonov's theorem ensures that there exists a unique solution of $\hat{\alpha}_I(t)$, $\hat{\alpha}(t)$, and $\hat{q}(t)$ that tracks $\alpha_{I_r}(t)$, $\alpha_r(t)$, and $q_r(t)$ on the order of ϵ :

$$\begin{bmatrix} \hat{\alpha}_I(t) \\ \hat{\alpha}(t) \\ \hat{q}(t) \end{bmatrix} = \begin{bmatrix} \alpha_{I_r}(t) \\ \alpha_r(t) \\ q_r(t) \end{bmatrix} + O(\epsilon) \quad (26)$$

uniformly in time. Thus, there exists a compact set $\Omega_{\hat{\alpha}}$ such that $\hat{\alpha} \in \Omega_{\hat{\alpha}}$ for all $t \geq 0$. Choosing $\Omega_{\alpha, \delta_e} = \Omega_{\hat{\alpha}} \times \Omega_{\delta_e}$ as the set of RBF distribution, standard Lyapunov arguments can be applied to prove that the projection based adaptation law in Eq. (20) ensures that the prediction error $e_s(t)$ remains uniformly ultimately bounded (UUB). Recalling that

$$\begin{aligned} [\alpha_{I_r}(t) \ \alpha_r(t) \ q_r(t)]^\top &= [\hat{\alpha}_I(t) \ \hat{\alpha}(t) \ \hat{q}(t)]^\top - e(t) \\ &= [\alpha_I(t) \ \alpha(t) \ q(t)]^\top + e_s(t) - e(t) \end{aligned}$$

we obtain that as $t \rightarrow \infty$, the tracking error $[\alpha_l(t) \ \alpha(t) \ q(t)]^\top - [\alpha_{l_r}(t) \ \alpha_r(t) \ q_r(t)]^\top$ is UUB.

F. Simulation Example: Adaptive Design for F-16 Short-Period Dynamics

For simulations, data are taken from [14], which contains aerodynamic data for the nonlinear F-16 model throughout an AOA range of $[-10 \text{ deg}, 45 \text{ deg}]$. The baseline model of the F-16 short-period dynamics calculated at the trimmed airspeed of 502 ft/s and angle of attack $\alpha = 2.11 \text{ deg}$ is

$$A_p = \begin{bmatrix} -1.0190 & 1 \\ 0.8223 & -1.0774 \end{bmatrix}, \quad b_p = \begin{bmatrix} 0 \\ -0.1756 \end{bmatrix}$$

which leads to open-loop system eigenvalues $\lambda_1 = -1.9554$ and $\lambda_2 = -0.1409$. The commanded reference input of interest to track is

$$y^{\text{cmd}}(t) = 0.95 \left(\frac{1}{1 + e^{t-8}} - \frac{0.5}{1 + e^{t-8}} + \frac{1}{1 + e^{t-20}} - e^{-0.2t} - 0.5 \right)$$

which represents an aggressive flight maneuver, reaching an angle of attack up to 35 deg. The Ricatti equation is solved with the weighting matrices:

$$Q = \begin{bmatrix} 25 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad R = 0.01$$

which leads to the following LQR gains: $k_{\text{LQR}} = [-50.0000 \ -39.5483 \ -15.9570]$. The reference model for the nominal values of the aircraft system is

$$\begin{bmatrix} \dot{\alpha}_{l_r}(t) \\ \dot{\alpha}_r(t) \\ \dot{q}_r(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1.0000 & 0 \\ 0 & -1.0189 & 1.0000 \\ -8.7800 & -6.1224 & -3.8795 \end{bmatrix}}_{A_r} \begin{bmatrix} \alpha_{l_r}(t) \\ \alpha_r(t) \\ q_r(t) \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} y^{\text{cmd}}(t)$$

having its closed-loop eigenvalues at $\lambda_{\text{LQR}} = -2.0863, -1.4060 \pm 1.4938i$. The plot in Fig. 4a shows the tracking result of the linear system without uncertainties for initial conditions $\alpha_{r_0} = 0 \text{ deg}, q_{r_0} = 0 \text{ deg}$, and $\alpha_{l_{r_0}} = \alpha_{r_0} - y^{\text{cmd}}(0)$. It is noted that the baseline controller has an infinite gain margin and a phase margin of 65.7621 deg at crossover frequency $\omega_{c_p} = 3.1094 \text{ rad/s}$, so that there is no high gain in the baseline design.

Next, consider the case when $f(\alpha, \delta_e)$ cannot be neglected and is unknown. Using the wind-tunnel data from [14] for $\beta = -30 \text{ deg}$ and curve-fitting methods, we simulate $f(\alpha, \delta_e)$ given by Eq. (5) using $\sigma = 0.25, C_0 = 0.1$, and $h = 0.14$. Figure 1 demonstrates the complete structure of $f(\alpha, \delta_e)$ as compared to the wind-tunnel data from [14]. Figure 4 shows that the PI controller can retain its tracking performance in the presence of uncertainties at the price of increased control effort. The nonlinearity $f(\alpha, \delta_e)$ causes the control effort to increase past 40 deg, well beyond the capabilities of typical elevator deflection limits. If the weighting matrix R is instead chosen to be $R = 7$, the control effort of the PI controller remains within elevator deflection limits, but the tracking performance is poor (Fig. 5).

To implement the adaptive controller, the state predictor is designed with 12 RBFs. Of these, four are $\Phi_1(\alpha)$ -type Gaussians evenly distributed over $\alpha \in [-45 \text{ deg}, 45 \text{ deg}]$ with $\delta = 1$. The

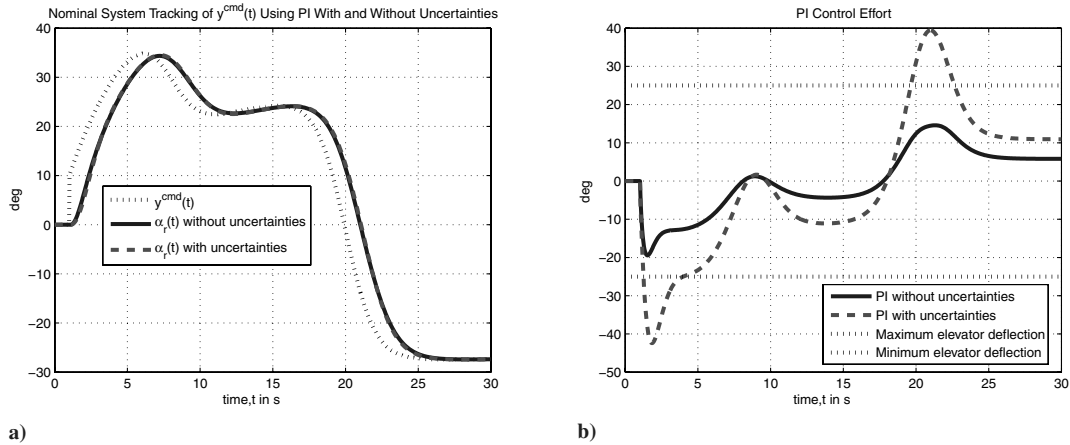


Fig. 4 System performance using the PI controller.

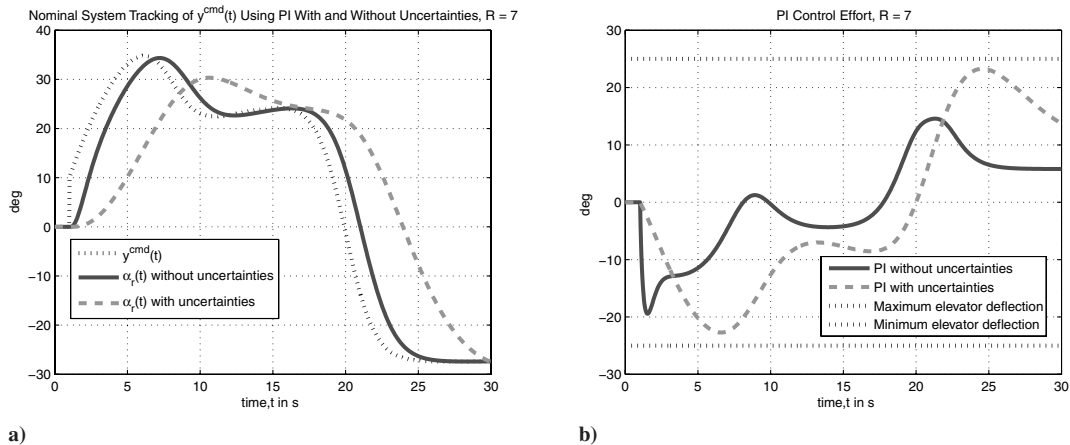


Fig. 5 System performance using the PI controller, $R = 7$.

remaining eight $\Phi_2(\alpha, \delta_e)$ -type basis functions are evenly distributed over $\alpha \in [-45^\circ, 45^\circ]$, $\delta_{e_{ad}} \in [-30^\circ, 30^\circ]$ with $\rho = 5$. The norm upper bound for the implementation of the projection operator is set to $W^* = 10$, the lower bound for the positive weights w is set to 0.01, and the adaptation gain is set to $\Gamma = 0.2$. The following state predictor is implemented:

$$\begin{bmatrix} \dot{\hat{\alpha}}_I(t) \\ \dot{\hat{\alpha}}(t) \\ \dot{\hat{q}}(t) \end{bmatrix} = A_r \begin{bmatrix} \hat{\alpha}_I(t) \\ \hat{\alpha}(t) \\ \hat{q}(t) \end{bmatrix} + b(-\delta_{e_{ad}}(t) + \hat{\theta}^\top(t)\Phi_1(\alpha(t)) + \hat{w}^\top(t)\Phi_2(\alpha(t), \delta_e(t)))$$

Then

$$\dot{e}(t) = A_r e(t) + b(-\delta_{e_{ad}}(t) + \hat{\theta}^\top(t)\Phi_1(\alpha(t)) + \hat{w}^\top(t)\Phi_2(\alpha(t), \delta_e(t)))$$

To obtain the desired system performance, the following equation needs to be approximately solved for adaptive elevator control, $\delta_{e_{ad}}$:

$$\delta_{e_{ad}}(t) = \hat{\theta}^\top(t)\Phi_1(\alpha(t)) + \hat{w}^\top(t)\Phi_2(\alpha(t), \delta_{e_{nom}}(t) - \delta_{e_{ad}}(t))$$

where $\delta_{e_{nom}}(t)$ is defined in Eq. (9). Obviously, this cannot be solved in terms of analytical functions. The fast dynamics are designed as

$$\begin{aligned} -0.3\dot{\delta}_{e_{ad}}(t) &= \delta_{e_{ad}}(t) - \hat{\theta}^\top(t)\Phi_1(\alpha(t)) \\ &- \hat{w}^\top(t)\Phi_2(\alpha(t), \delta_{e_{nom}}(t) - \delta_{e_{ad}}(t)) \end{aligned}$$

The plots are given in Figs. 6 and 7. Figure 6a shows the closed-loop tracking performance of the estimator state $\hat{\alpha}(t)$ to the commanded reference input $y^{\text{cmd}}(t)$ and the actual state $\alpha(t)$ to the reference state $\alpha_r(t)$, and Fig. 6b shows the closed-loop tracking performance of the estimator state $\hat{q}(t)$ to the reference state $q_r(t)$ and the actual state $q(t)$ to the reference state $q_r(t)$. We note that all the states and control input remain in the domain of the RBF approximation. Figure 7a shows the adaptive control effort $\delta_{e_{ad}}(t)$, which closely matches the unknown nonlinearity, $f(\alpha(t), \delta_e(t))$, as a function of time. Finally, Fig. 7b shows that the control effort using adaptation is smaller in magnitude when compared to the performance of the PI controller in the presence of uncertainties. It can be seen that the control effort using adaptation stays within physical limits of the elevator deflection.

III. Adaptive Control Design for Nonlinear-In-Control Multi-Input Aircraft System

A. Aircraft Roll/Yaw Dynamics

Using a 6-degree of freedom (DOF) flat-Earth, body-axis aircraft model, the kinematic Euler roll rate equation is known to be

$$\dot{\phi} = p + \tan \theta (q \sin \phi + r \cos \phi) \quad (27)$$

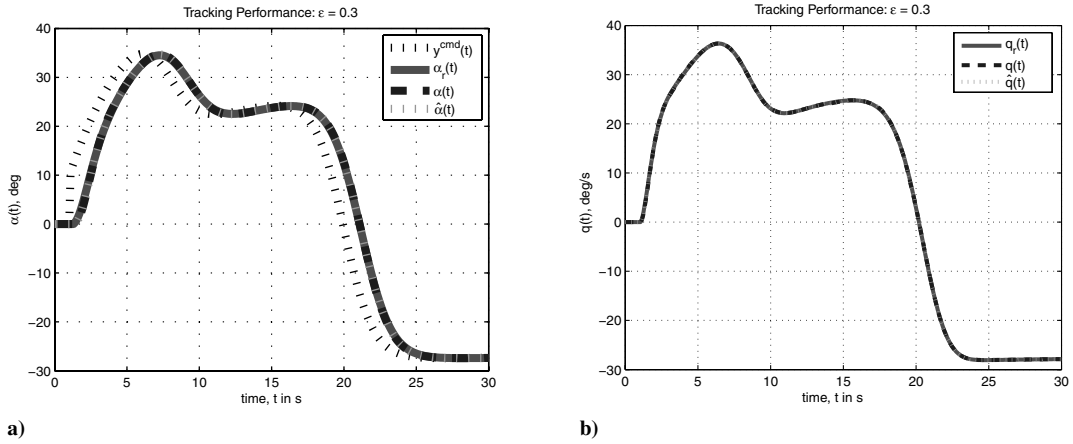
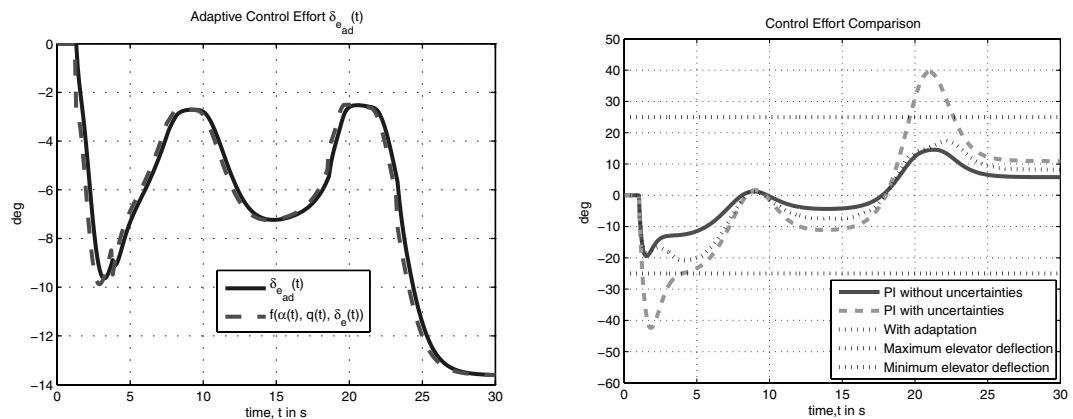


Fig. 6 System performance.



a) Adaptive elevator control, $\delta_{e_{ad}}(t)$ compared to the unknown nonlinearity $f(\alpha(t), \delta_e(t))$

b) Control efforts: PI with no uncertainties and adaptive with uncertainties are within the control limits, while baseline PI with uncertainties is outside the elevator actuator limits

Fig. 7 Control efforts.

where p , q , and r are the body-axis roll, pitch, and yaw rates, respectively, θ is the pitch angle, and ϕ is the roll angle [14]. Let the trimmed pitch angle be θ_0 . For small roll angle ϕ , the following holds:

$$\dot{\phi} = p + r \tan \theta_0 \quad (28)$$

For a trimmed angle of attack α_0 , one can write

$$p_s = p \cos \alpha_0 + r \sin \alpha_0 \quad r_s = r \cos \alpha_0 - p \sin \alpha_0 \quad (29)$$

where p_s and r_s are the stability axis roll and yaw rates. Solving Eq. (29) for p and r and directly substituting into Eq. (28) yields

$$\dot{\phi} = (\cos \alpha_0 + \sin \alpha_0 \tan \theta_0) p_s + (\cos \alpha_0 \tan \theta_0 - \sin \alpha_0) r_s \quad (30)$$

An additional relationship between flight path angle γ , pitch angle, and angle of attack at zero bank angle and zero angle of sideslip (AOS) is

$$\alpha_0 = \theta_0 - \gamma_0$$

which can be substituted into Eq. (30) to get

$$\dot{\phi} = \frac{\cos \gamma_0}{\cos \theta_0} p_s + \frac{\sin \gamma_0}{\cos \theta_0} r_s \quad (31)$$

The angle of sideslip dynamics in a stability axis assuming small angles has the form

$$\dot{\beta} = \frac{1}{V} (Y_\beta \beta + Y_r r_s + Y_{\delta_a} \delta_a + Y_{\delta_r} \delta_r) + \left(\frac{g \cos \theta_0}{V} \right) \phi - r_s$$

where V is the true airspeed, g is the gravitational constant, and δ_a and δ_r are aileron and rudder control. The terms Y_β , Y_r , Y_{δ_a} , and Y_{δ_r} are aerodynamic stability and control derivatives that change slowly with time and can be adequately approximated by constants. Additionally, Y_{δ_a} and Y_{δ_r} are known to be small with respect to true airspeed V , so that for control design purposes,

$$\frac{Y_{\delta_a}}{V} \approx \frac{Y_{\delta_r}}{V} \approx 0$$

The full roll/yaw dynamics expressed in the stability axis are

$$\begin{aligned} \dot{\phi} &= \frac{\cos \gamma_0}{\cos \theta_0} p_s + \frac{\sin \gamma_0}{\cos \theta_0} r_s \quad \dot{\beta} = \frac{Y_\beta}{V} \beta + \frac{Y_r}{V} r_s + \frac{g \cos \theta_0}{V} \phi - r_s \\ \dot{p}_s &= L_\beta \beta + L_p p_s + L_r r_s + \delta_l(p_s, r_s) + L_{\delta_a}(\beta, \delta_a) + L_{\delta_r}(\beta, \delta_r) \\ \dot{r}_s &= N_\beta \beta + N_p p_s + N_r r_s + \delta_n(p_s, r_s) + N_{\delta_a}(\beta, \delta_a) + N_{\delta_r}(\beta, \delta_r) \end{aligned} \quad (32)$$

where δ_l and δ_n are incremental rolling and yawing moments that are generally *unknown* functions of the rates p_s , q_s , and r_s . Additionally, L_{δ_a} , L_{δ_r} , N_{δ_a} , and N_{δ_r} are rolling and yaw moments due to aileron and rudder deflections that are also *unknown*. However, some partial knowledge is assumed from flight tests so that L_{δ_a} , L_{δ_r} , N_{δ_a} , and N_{δ_r} can be decomposed into a known (nominal) linear part and an unknown nonlinear part as follows:

$$\begin{aligned} L_{\delta_a}(\beta, \delta_a) &= L_{\delta_{a0}} \delta_a + L_{\delta_a}(\beta, \delta_a) \\ L_{\delta_r}(\beta, \delta_r) &= L_{\delta_{r0}} \delta_r + L_{\delta_r}(\beta, \delta_r) \\ N_{\delta_a}(\beta, \delta_a) &= N_{\delta_{a0}} \delta_a + N_{\delta_a}(\beta, \delta_a) \\ N_{\delta_r}(\beta, \delta_r) &= N_{\delta_{r0}} \delta_r + N_{\delta_r}(\beta, \delta_r) \end{aligned} \quad (33)$$

Additionally, the nonlinear expressions $\delta_l(p_s, r_s) + L_{\delta_a}(\beta, \delta_a) + L_{\delta_r}(\beta, \delta_r)$ and $\delta_n(p_s, r_s) + N_{\delta_a}(\beta, \delta_a) + N_{\delta_r}(\beta, \delta_r)$ can be expressed as linear combinations of two unknown functions of β , p_s , r_s , δ_a , and δ_r . Substituting Eq. (33), we get that

$$\begin{aligned} &\delta_l(p_s, r_s) + L_{\delta_a}(\beta, \delta_a) + L_{\delta_r}(\beta, \delta_r) \\ &= L_{\delta_{a0}}(\delta_a + f_1(\beta, p_s, r_s, \delta_a)) + L_{\delta_{r0}}(\delta_r + f_2(\beta, p_s, r_s, \delta_r)) \\ &\delta_n(p_s, r_s) + N_{\delta_a}(\beta, \delta_a) + N_{\delta_r}(\beta, \delta_r) \\ &= N_{\delta_{a0}}(\delta_a + f_1(\beta, p_s, r_s, \delta_a)) + N_{\delta_{r0}}(\delta_r + f_2(\beta, p_s, r_s, \delta_r)) \end{aligned}$$

Substituting this expression into Eq. (32) and rewriting the dynamics in state space form gives

$$\begin{aligned} \begin{bmatrix} \dot{\beta}(t) \\ \dot{\phi}(t) \\ \dot{p}_s(t) \\ \dot{r}_s(t) \end{bmatrix} &= A_p \underbrace{\begin{bmatrix} \beta(t) \\ \phi(t) \\ p_s(t) \\ r_s(t) \end{bmatrix}}_{x(t)} \\ &+ B_p \begin{bmatrix} \delta_a(t) + f_1(\beta(t), p_s(t), r_s(t), \delta_a(t)) \\ \delta_r(t) + f_2(\beta(t), p_s(t), r_s(t), \delta_r(t)) \end{bmatrix} \\ y(t) &= \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}}_{C_p^T} \begin{bmatrix} \beta(t) \\ \phi(t) \\ p_s(t) \\ r_s(t) \end{bmatrix} \end{aligned} \quad (34)$$

where

$$A_p = \begin{bmatrix} \frac{Y_\beta}{V} & \frac{g \cos \theta_0}{V} & 0 & \frac{Y_r - 1}{V} \\ 0 & 0 & \frac{\cos \gamma_0}{\cos \theta_0} & \frac{\sin \gamma_0}{\cos \theta_0} \\ L_\beta & 0 & L_p & L_r \\ N_\beta & 0 & N_p & N_r \end{bmatrix}, \quad B_p = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ L_{\delta_{a0}} & L_{\delta_{r0}} \\ N_{\delta_{a0}} & N_{\delta_{r0}} \end{bmatrix}$$

and C_p^T is the output matrix that defines the outputs of interest for tracking. The controllability matrix has rank = 4, so the system is controllable. The control objective is to design a control signal

$$\delta(t) = \begin{bmatrix} \delta_a(t) \\ \delta_r(t) \end{bmatrix} = \begin{bmatrix} \delta_{a_{nom}}(t) - \delta_{a_{ad}}(t) \\ \delta_{r_{nom}}(t) - \delta_{r_{ad}}(t) \end{bmatrix} \quad (35)$$

such that the angles β and ϕ track a smooth commanded reference input $Y^{cmd}(t)$ in the presence of roll/yaw parametric uncertainties and aileron/rudder nonlinear unknown control effects.

Again, based on wind-tunnel data taken from [14] for high angle flight regimes ($\alpha = 45^\circ$) and curve-fitting methods, the unknown nonlinear terms f_1 and f_2 can be approximated by the following functions:

$$\begin{aligned} f_1(\beta, p_s, r_s, \delta_a) &= ((1 - C_1) e^{-\frac{(\beta - \beta_0)^2}{2\sigma_1^2}} + C_1) (\tanh(\delta_a + h_1) \\ &+ \tanh(\delta_a - h_1) + 0.001\delta_a) \\ &+ D_1 \cos(A_1 p_s - \omega_1) \sin(A_2 r_s - \omega_2) + D_2 \end{aligned} \quad (36)$$

$$\begin{aligned} f_2(\beta, p_s, r_s, \delta_r) &= ((1 - C_2) e^{-\frac{(\beta - \beta_0)^2}{2\sigma_2^2}} + C_2) (\tanh(\delta_r + h_2) \\ &+ \tanh(\delta_r - h_2) + 0.001\delta_r) \\ &+ D_3 \cos(A_3 p_s - \omega_3) \sin(A_4 r_s - \omega_4) + D_4 \end{aligned} \quad (37)$$

The positive constants C_1 and C_2 represent the percentage of control effectiveness available at high angles of sideslip and are always less than unity. When the system is at a trim sideslip angle, full aileron and rudder effectiveness are obtained. The parameters σ_1 and σ_2 define the widths of the Gaussian functions $e^{-(\beta - \beta_0)^2 / 2\sigma_1^2}$ and $e^{-(\beta - \beta_0)^2 / 2\sigma_2^2}$. An additional small aileron and rudder control-induced moment terms are introduced so that for small control efforts, these surfaces are within a boundary layer and produce negligible aerodynamic moments. As the control effort increases, the surfaces stall and fail to produce any additional moment and the system reaches saturation, which is approximated by the hyperbolic tangent functions where h_1 and h_2 are constants. Last, the rolling and yaw

moments can be locally approximated by sine functions where the amplitude and phase are designed based on wind-tunnel data analysis.

B. Nominal Roll/Yaw Model

The linear-in-control reference system augmented with the integral states $\beta_I(t)$ and $\phi_I(t)$ for the roll/yaw dynamics of Eq. (34) in the absence of uncertainties is

$$\begin{bmatrix} \dot{\beta}_I(t) \\ \dot{\phi}_I(t) \\ \dot{\beta}_r(t) \\ \dot{\phi}_r(t) \\ \dot{p}_{s_r}(t) \\ \dot{r}_{s_r}(t) \end{bmatrix} = A_r \underbrace{\begin{bmatrix} \beta_I(t) \\ \phi_I(t) \\ \beta_r(t) \\ \phi_r(t) \\ p_{s_r}(t) \\ r_{s_r}(t) \end{bmatrix}}_{x_r(t)} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} Y^{\text{cmd}}(t) \quad (38)$$

where A_r is Hurwitz (all eigenvalues of A_r have a strictly negative real part) and is computed from standard LQR methods. Again, the reference model dynamics (38) is chosen to represent the nominal system operating under the baseline LQR PI controller. The nominal controller is given by

$$\begin{bmatrix} \delta_{a_{\text{nom}}}(t) \\ \delta_{r_{\text{nom}}}(t) \end{bmatrix} = -K_{\text{LQR}}^\top x_r(t) \quad (39)$$

where K_{LQR} is a matrix of constant gains, which can be substituted into the roll/yaw dynamics (34) to give

$$\begin{bmatrix} \dot{\beta}_I(t) \\ \dot{\phi}_I(t) \\ \dot{\beta}(t) \\ \dot{\phi}(t) \\ \dot{p}_s(t) \\ \dot{r}_s(t) \end{bmatrix} = A_r \underbrace{\begin{bmatrix} \beta_I(t) \\ \phi_I(t) \\ \beta(t) \\ \phi(t) \\ p_s(t) \\ r_s(t) \end{bmatrix}}_{x(t)} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} Y^{\text{cmd}}(t) \\ + B \begin{bmatrix} -\delta_{a_{\text{ad}}}(t) + f_1(\beta(t), p_s(t), r_s(t), \delta_{a_{\text{nom}}}(t) - \delta_{a_{\text{ad}}}(t)) \\ -\delta_{r_{\text{ad}}}(t) + f_2(\beta(t), p_s(t), r_s(t), \delta_{r_{\text{nom}}}(t) - \delta_{r_{\text{ad}}}(t)) \end{bmatrix} \\ y(t) = [0 \ C_p]x(t) \quad (40)$$

where

$$B = \begin{bmatrix} 0 \\ B_p \end{bmatrix}$$

C. RBF Approximations for Multi-Input System

From Eq. (40), the unknown nonlinearities of the system do not depend explicitly on t , and therefore can be parameterized using RBF neural networks. Again, using the Leibnitz formula and the main results of [15], the nonlinearities f_1 and f_2 can be represented over a compact set of initial conditions of interest as

$$f_1(\beta, p_s, r_s, \delta_a) = W_1^\top \Phi_1(\beta, p_s, r_s, \delta_a) + \varepsilon_1(\beta, p_s, r_s, \delta_a) \\ |\varepsilon_1(\beta, p_s, r_s, \delta_a)| \leq \varepsilon_1^* \\ f_2(\beta, p_s, r_s, \delta_r) = W_2^\top \Phi_2(\beta, p_s, r_s, \delta_r) + \varepsilon_2(\beta, p_s, r_s, \delta_r) \\ |\varepsilon_2(\beta, p_s, r_s, \delta_r)| \leq \varepsilon_2^*$$

where

$$W_1 = [\theta_1^\top \ w_1^\top]^\top = [\theta_{1,1} \cdots \theta_{1,M_1} \ w_{1,1} \cdots w_{1,N_1}]^\top \\ W_2 = [\theta_2^\top \ w_2^\top]^\top = [\theta_{2,1} \cdots \theta_{2,M_2} \ w_{2,1} \cdots w_{2,N_2}]^\top \\ \Phi_1(\beta, p_s, r_s, \delta_a) = [\Phi_{1,1}^\top(\beta, p_s, r_s) \ \Phi_{1,2}^\top(\beta, p_s, r_s, \delta_a)]^\top \\ \Phi_2(\beta, p_s, r_s, \delta_r) = [\Phi_{2,1}^\top(\beta, p_s, r_s) \ \Phi_{2,2}^\top(\beta, p_s, r_s, \delta_r)]^\top$$

with $\Phi_{1,1}(\beta, p_s, r_s)$ and $\Phi_{2,1}(\beta, p_s, r_s)$ as radial basis functions over the sets $(\beta, p_s, r_s, \delta_a) \in \Omega_\beta \times \Omega_{p_s} \times \Omega_{r_s} \times \Omega_{\delta_a}$ and $(\beta, p_s, r_s, \delta_r) \in \Omega_\beta \times \Omega_{p_s} \times \Omega_{r_s} \times \Omega_{\delta_r}$.

D. Roll/Yaw Dynamics State Predictor

Consider the following state predictor for the dynamics in Eq. (34):

$$\begin{bmatrix} \hat{\beta}_I(t) \\ \hat{\phi}_I(t) \\ \hat{\beta}(t) \\ \hat{\phi}(t) \\ \hat{p}_s(t) \\ \hat{r}_s(t) \end{bmatrix} = A_r \underbrace{\begin{bmatrix} \hat{\beta}_I(t) \\ \hat{\phi}_I(t) \\ \hat{\beta}(t) \\ \hat{\phi}(t) \\ \hat{p}_s(t) \\ \hat{r}_s(t) \end{bmatrix}}_{\hat{x}(t)} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} Y^{\text{cmd}}(t) \\ + B \begin{bmatrix} -\delta_{a_{\text{ad}}} + \hat{W}_1^\top(t) \Phi_1(\beta(t), p_s(t), r_s(t), \delta_a(t)) \\ -\delta_{r_{\text{ad}}} + \hat{W}_2^\top(t) \Phi_2(\beta(t), p_s(t), r_s(t), \delta_r(t)) \end{bmatrix} \quad (41)$$

where \hat{W}_1 and \hat{W}_2 are the estimates of the unknown constant vectors W_1 and W_2 . Define the prediction error signal as

$$e_s(t) = \hat{x}(t) - x(t) \quad (42)$$

The prediction error dynamics are

$$\dot{e}_s(t) = A_r e_s(t) \\ + B \begin{bmatrix} \tilde{W}_1^\top(t) \Phi_1(\beta(t), p_s(t), r_s(t), \delta_a(t)) - \varepsilon_1(\beta(t), p_s(t), r_s(t), \delta_a(t)) \\ \tilde{W}_2^\top(t) \Phi_2(\beta(t), p_s(t), r_s(t), \delta_r(t)) - \varepsilon_2(\beta(t), p_s(t), r_s(t), \delta_r(t)) \end{bmatrix} \\ = A_r e_s(t) + B_1(\tilde{W}_1^\top(t) \Phi_1(\beta(t), p_s(t), r_s(t), \delta_a(t)) \\ - \varepsilon_1(\beta(t), p_s(t), r_s(t), \delta_a(t))) \\ + B_2(\tilde{W}_2^\top(t) \Phi_2(\beta(t), p_s(t), r_s(t), \delta_r(t)) \\ - \varepsilon_2(\beta(t), p_s(t), r_s(t), \delta_r(t))) \quad (43)$$

with the parameter errors $\tilde{W}_1(t) = \hat{W}_1(t) - W_1(t)$, $\tilde{W}_2(t) = \hat{W}_2(t) - W_2(t)$ and B_1 and B_2 denoting the first and second columns of B , respectively. With these definitions, the adaptive law

$$\dot{\hat{W}}_1(t) = \Gamma_1 \text{Proj}(\hat{W}_1(t), -\Phi_1(\beta(t), p_s(t), r_s(t), \delta_a(t)) e_s^\top(t) P_0 B_1) \\ \dot{\hat{W}}_2(t) = \Gamma_2 \text{Proj}(\hat{W}_2(t), -\Phi_2(\beta(t), p_s(t), r_s(t), \delta_r(t)) e_s^\top(t) P_0 B_2) \quad (44)$$

where $\text{Proj}(\cdot, \cdot)$ denotes the projection operator [15], $P_0 = P_0^\top > 0$ solves the Lyapunov equation $A_r^\top P_0 + P_0 A_r = -Q_0$ for a matrix $Q_0 > 0$, and $\Gamma_1^\top = \Gamma_1 > 0$ and $\Gamma_2^\top = \Gamma_2 > 0$ are matrices of adaptation gains ensure that the parameter errors $\tilde{W}_1(t)$ and $\tilde{W}_2(t)$ are bounded. The proof can be found in [18].

E. MIMO Control Design

Let the tracking error signal between the predictor and the reference system be

$$e(t) = \hat{x}(t) - x_r(t) \quad (45)$$

The open-loop time-varying tracking error dynamics are

$$\dot{e}(t) = A_r e(t) + B \begin{bmatrix} -\delta_{a_{ad}}(t) + \hat{W}_1^\top(t) \Phi_1(\beta(t), p_s(t), r_s(t), \delta_a(t)) \\ -\delta_{r_{ad}}(t) + \hat{W}_2^\top(t) \Phi_2(\beta(t), p_s(t), r_s(t), \delta_r(t)) \end{bmatrix} \quad (46)$$

A dynamic inversion based controller is to be determined from the solution of the following system of equations:

$$\begin{bmatrix} \delta_{a_{ad}} \\ \delta_{r_{ad}} \end{bmatrix} = \begin{bmatrix} \hat{W}_1^\top \Phi_1(\beta, p_s, r_s, \delta_{a_{nom}} - \delta_{a_{ad}}(t)) \\ \hat{W}_2^\top \Phi_2(\beta, p_s, r_s, \delta_{r_{nom}} - \delta_{r_{ad}}(t)) \end{bmatrix} \quad (47)$$

which would result in the stable closed-loop tracking error dynamics $\dot{e}(t) = A_r e(t)$. In general, Eq. (47) cannot be solved explicitly for $\delta_{a_{ad}}$ and $\delta_{r_{ad}}$, so an approximation of the dynamic inversion controller is constructed via fast dynamics:

$$\epsilon \begin{bmatrix} \dot{\delta}_{a_{ad}}(t) \\ \dot{\delta}_{r_{ad}}(t) \end{bmatrix} = -\bar{\mathbf{P}}(t, e(t), \delta_{a_{ad}}(t), \delta_{r_{ad}}(t)) \mathbf{f}(t, e(t), \delta_{a_{ad}}(t), \delta_{r_{ad}}(t)), \quad 0 < \epsilon \ll 1 \quad (48)$$

where

$$\begin{aligned} \mathbf{f}(t, e, \delta_{a_{ad}}, \delta_{r_{ad}}) &= \begin{bmatrix} \delta_{a_{ad}} - \hat{W}_1^\top(t) \Phi_1(\beta(t), p_s(t), r_s(t), \delta_{a_{nom}}(t) - \delta_{a_{ad}}(t)) \\ \delta_{r_{ad}} - \hat{W}_2^\top(t) \Phi_2(\beta(t), p_s(t), r_s(t), \delta_{r_{nom}}(t) - \delta_{r_{ad}}(t)) \end{bmatrix} \end{aligned} \quad (49)$$

and

$$\begin{aligned} \bar{\mathbf{P}}(t, e, \delta_{a_{ad}}, \delta_{r_{ad}}) &= \text{diag} \left\{ \begin{aligned} &-\hat{W}_1^\top(t) \frac{\partial \Phi_1}{\partial \delta_{a_{ad}}}(\beta(t), p_s(t), r_s(t), \delta_{a_{ad}}(t)) + 1 \\ &-\hat{W}_2^\top(t) \frac{\partial \Phi_2}{\partial \delta_{r_{ad}}}(\beta(t), p_s(t), r_s(t), \delta_{r_{ad}}(t)) + 1 \end{aligned} \right\} \end{aligned}$$

is the Jacobian matrix of $\mathbf{f}(t, e, \delta_{a_{ad}}, \delta_{r_{ad}})$. The existence of a root for Eq. (49) within a neighborhood can be shown by noting that the Jacobian matrix $\bar{\mathbf{P}}$ is always full rank and applying the implicit function theorem of [17]. Let $h(t, e, \delta_a, \delta_r)$ denote this isolated root of $\mathbf{f}(t, e, \delta_a, \delta_r)$ and define

$$\tau = \frac{t}{\epsilon}, \quad v(t, \beta) = \begin{bmatrix} \delta_{a_{ad}}(t) \\ \delta_{r_{ad}}(t) \end{bmatrix} - h(t, \beta) \quad (50)$$

The *reduced system* for Eqs. (45), (46), and (48) is given by

$$\dot{e}(t) = A_r e(t), \quad e(0) = e_0 \quad (51)$$

and the *boundary layer system* is

$$\begin{aligned} \frac{dv}{d\tau} &= -\bar{\mathbf{P}}(t, e, v + h(t, e)) \mathbf{f}(t, e, v + h(t, e)) \\ v(0) &\triangleq v_0 = \begin{bmatrix} \delta_{a_0} \\ \delta_{r_0} \end{bmatrix} - h(t, e_0) \end{aligned} \quad (52)$$

Then, following [12], the boundary layer system (52) has exponentially stable origin. Moreover, the system given by Eqs. (41) and (48) has a unique solution

$$\hat{x}(t) = x_r(t) + O(\epsilon) \quad (53)$$

uniformly in t . The proof is straightforward and is derived by finding a time-varying Lyapunov function which satisfies the conditions on exponential stability of a nonautonomous system and is given in [18]. It follows that there exists a compact set $\Omega_{\hat{\beta}}$ such that $\hat{\beta} \in \Omega_{\hat{\beta}}$ for all $t \geq 0$. We choose the set of RBF distribution to be $\Omega_{\hat{\beta}} \times \Omega_{p_s} \times \Omega_{r_s} \times \Omega_{\delta_a} \times \Omega_{\delta_r}$ and apply standard Lyapunov arguments to conclude that the projection based adaptive law defined in Eq. (44) ensures that the prediction error $e_s(t)$ is bounded. Hence, following the same logic as in the short-period case, we see that the tracking error $x(t) - x_r(t)$ is UUB as $t \rightarrow \infty$.

F. Example: Adaptive Control Design for F-16 Roll/Yaw Dynamics

For our simulations, we use the lateral/directional baseline model of an F-16 from [14] flying at sea level with an airspeed of 502 ft/s and angle of attack of $\alpha = 2.11$ deg. The model matrices are given as

$$A_p = \begin{bmatrix} -0.3220 & 0.0640 & 0.0364 & -0.9917 \\ 0 & 0 & 1 & 0.0393 \\ -30.6490 & 0 & -3.6784 & 0.6646 \\ 8.5395 & 0 & -0.0254 & -0.4764 \end{bmatrix}$$

$$B_p = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -0.7331 & 0.1315 \\ -0.0319 & -0.0620 \end{bmatrix}$$

and its open-loop eigenvalues are

$$\lambda_1 = -3.6153 \quad \lambda_{2,3} = -0.4237 \pm 3.0635i \quad \lambda_4 = -0.0142$$

Note that the roll/yaw system is stable, but its eigenvalues imply low damping. Next we select our reference model by solving the Riccati equation with the following weighted matrices:

$$Q = \begin{bmatrix} 75 & 0 & 0 & 0 & 0 & 0 \\ 0 & 25 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad R = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.001 \end{bmatrix}$$

so that

$$K_{LQR} = \begin{bmatrix} 17.1604 & -49.0086 & 35.0149 & -31.2757 & -5.2083 & -11.7945 \\ 268.4310 & 31.3305 & 67.8590 & 8.1376 & 1.1945 & -41.9878 \end{bmatrix}$$

and

$$A_r = \begin{bmatrix} 0 & 0 & 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0000 & 0 & 0 \\ 0 & 0 & -0.3220 & 0.0640 & 0.0364 & -0.9917 \\ 0 & 0 & 0 & 0 & 1.0000 & 0.0393 \\ -22.7184 & -40.0481 & -13.9032 & -23.9983 & -7.6537 & -2.4606 \\ 17.1901 & 0.3791 & 13.8638 & -0.4932 & -0.1175 & -3.4559 \end{bmatrix}$$

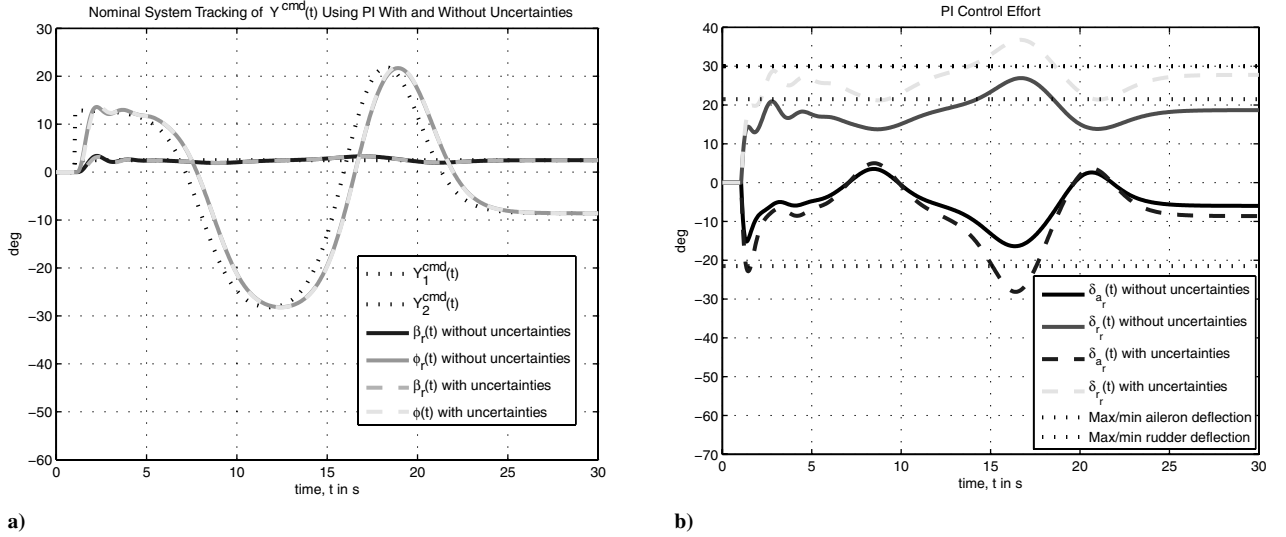


Fig. 8 System performance using the PI controller.

The eigenvalues of the reference model are

$$\begin{aligned}\lambda_1 &= -4.2216 & \lambda_{2,3} &= -1.0810 \pm 3.4774i \\ \lambda_{4,5} &= -1.7806 \pm 2.2150i & \lambda_6 &= -1.4967\end{aligned}$$

The commanded reference input of interest to track is

$$Y^{\text{cmd}}(t) = \begin{bmatrix} 0.0436 \\ 0.75 \left(\frac{1}{1+e^{t-8}} - \frac{1.5}{1+e^{t-16}} + \frac{1}{1+e^{t-20}} - 0.2 \right) \end{bmatrix}$$

where the β command is a step input of magnitude 2.5 deg that starts after 1 s. In the absence of uncertainties, the nominal system behaves optimally using the PI controller Fig. 8a. The gain crossover frequencies are 4.53 rad/s for the aileron–aileron loop and 4.36 rad/s for the rudder–rudder loop. Hence, there is no high gain in the baseline control design. Initial conditions used for the nominal system are $\beta_{r0} = 0$ deg, $\phi_{r0} = 0$ deg, $p_{s0} = 0$ deg/s, and $r_{s0} = 0$ deg/s. To insert uncertainties into the roll/yaw dynamics, explicit expressions must be found for the nonlinearities $f_1(\beta, p_s, r_s, \delta_a)$ and $f_2(\beta, p_s, r_s, \delta_r)$ introduced in Eqs. (36) and (37) for simulation purposes. Toward that end, the values of the constants $C_1, C_2, \sigma_1, \sigma_2$ and D_i, A_i, ω_i , for $i = \{1, 2, 3, 4\}$, are found by analyzing wind-tunnel data from [14] at angle of attack $\alpha = 45$ deg. Using this data set, curve-fitting methods can be applied to find approximations that are of the form (36) and (37). Figure 9 shows the approximations of $f_1(\beta, p_s, r_s, \delta_a)$ and $f_2(\beta, p_s, r_s, \delta_r)$ along with the wind-tunnel data with the constant parameters chosen as

$$\begin{aligned}A_1 &= 0.33 & A_2 &= 0.195 & A_3 &= 0.45 & A_4 &= 1.85 \\ D_1 &= 0.295 & D_2 &= -0.0865 & D_3 &= 0.055 \\ D_4 &= -0.007 & \omega_1 &= 1.6 & \omega_2 &= 0 & \omega_3 &= -1.9 \\ \omega_4 &= 0 & C_1 &= 0.3 & C_3 &= 0.3 & h_1 &= 7 & h_2 &= 2.7 \\ \sigma_1 &= 0.25 & \sigma_2 &= 0.25\end{aligned}$$

Physically, this means that at a high sideslip angle, only 30% of the control is available to the system. The curve-fit approximation assumes $\beta \in [-30 \text{ deg}, 30 \text{ deg}]$, $p_s \in [-360 \text{ deg}, 360 \text{ deg}]$, $r_s \in [-90 \text{ deg}, 90 \text{ deg}]$, $\delta_a \in [-21.5 \text{ deg}, 21.5 \text{ deg}]$, and $\delta_r \in [-30 \text{ deg}, 30 \text{ deg}]$. Figure 8 shows the performance of the PI controller when the uncertainties f_1 and f_2 are present in the system. The uncertainties $f_1(\beta, p_s, r_s, \delta_a)$ and $f_2(\beta, p_s, r_s, \delta_r)$ lead to an increase in $\delta_a(t)$ up to 28 deg and an increase in $\delta_r(t)$ of up to 36 deg, which is beyond the capabilities of any rudder and aileron surfaces, respectively. If the weighting gain R is increased so that $R = \text{diag}(3, 0.2)$, the control effort stays within physical limits of aileron deflection, but the tracking performance is degraded (Fig. 10). Hence, the motivation is for adaptive control to compensate for these uncertainties without implying large control efforts.

The state predictor is designed with two sets of 24 RBFs. For each of the sets, eight are $\Phi_{i,1}(\beta, p_s, r_s)$ -type Gaussians evenly distributed over $\beta \in [-30 \text{ deg}, 30 \text{ deg}]$, $p_s \in [-360 \text{ deg}, 360 \text{ deg}]$, $r_s \in [-90 \text{ deg}, 90 \text{ deg}]$ with width $\delta_1, \delta_2 = 1$ for $i = \{1, 2\}$. The remaining 16 were $\Phi_{1,2}(\beta, p_s, r_s, \delta_a)$, $\Phi_{2,2}(\beta, p_s, r_s, \delta_r)$ -type basis functions evenly distributed over $\beta \in [-30 \text{ deg}, 30 \text{ deg}]$, $p_s \in [-360 \text{ deg}, 360 \text{ deg}]$, $r_s \in [-90 \text{ deg}, 90 \text{ deg}]$, $\delta_{a,ad} \in [-21.5 \text{ deg}, 21.5 \text{ deg}]$, and $\delta_{r,ad} \in [-30 \text{ deg}, 30 \text{ deg}]$ with lower $\rho_1 = \rho_2 = 5$. The norm upper bound is $W_1^* = W_2^* = 10$, the lower bound for the positive weights w_1 and w_2 is 0.01, and the adaptation gain is $\Gamma_1 = \Gamma_2 = 0.2$. The state predictor is

$$\dot{\hat{x}}(t) = A_r \hat{x}(t) + \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} Y^{\text{cmd}}(t) + B \begin{bmatrix} -\delta_{a,ad}(t) + \hat{\theta}_1^\top(t) \Phi_{1,1}(\beta(t), p_s(t), r_s(t)) + \hat{w}_1^\top(t) \Phi_{1,2}(\beta(t), p_s(t), r_s(t), \delta_a(t)) \\ -\delta_{r,ad}(t) + \hat{\theta}_2^\top(t) \Phi_{2,1}(\beta(t), p_s(t), r_s(t)) + \hat{w}_2^\top(t) \Phi_{2,2}(\beta(t), p_s(t), r_s(t), \delta_r(t)) \end{bmatrix}$$

and the error dynamics are

$$\dot{e}(t) = A_r e(t) + B \begin{bmatrix} -\delta_{a,ad}(t) + \hat{\theta}_1^\top(t) \Phi_{1,1}(\beta(t), p_s(t), r_s(t)) + \hat{w}_1^\top(t) \Phi_{1,2}(\beta(t), p_s(t), r_s(t), \delta_a(t)) \\ -\delta_{r,ad}(t) + \hat{\theta}_2^\top(t) \Phi_{2,1}(\beta(t), p_s(t), r_s(t)) + \hat{w}_2^\top(t) \Phi_{2,2}(\beta(t), p_s(t), r_s(t), \delta_r(t)) \end{bmatrix}$$

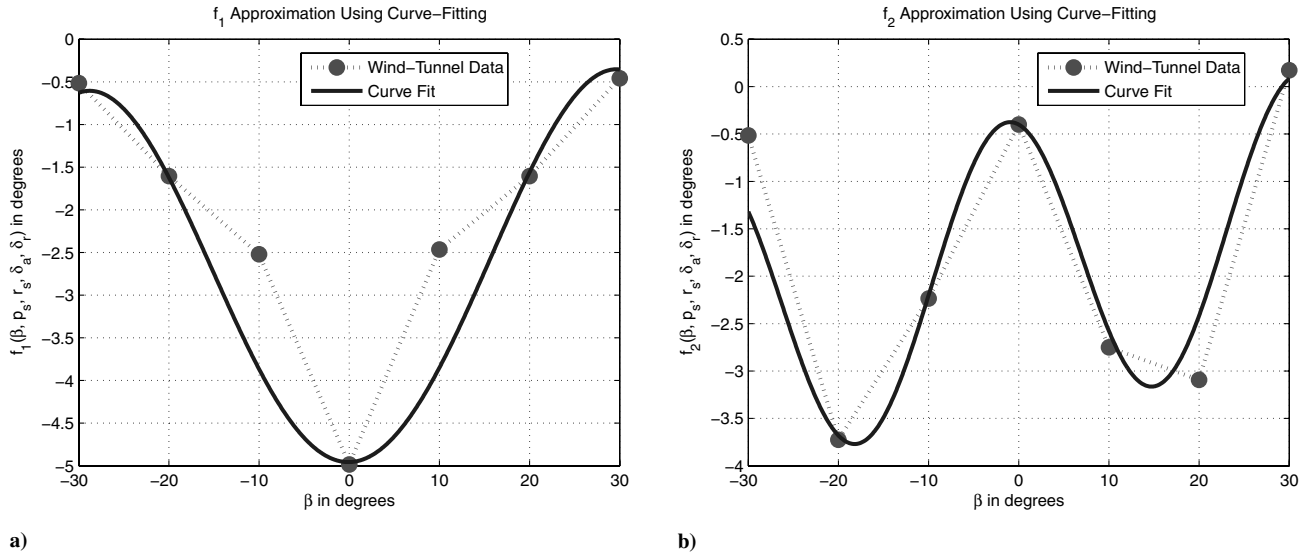
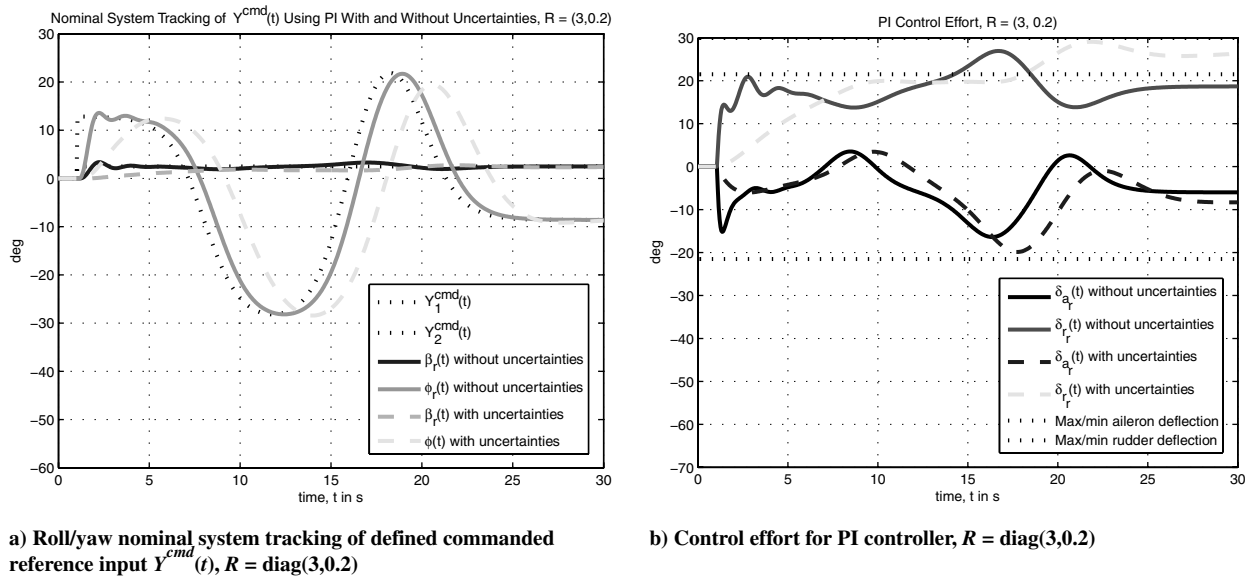


Fig. 9 Wind-tunnel data and curve fitting.

Fig. 10 System performance using the PI controller, $R = \text{diag}(3, 3.02)$.

To achieve the desired performance, the following equation needs to be approximately solved online for adaptive aileron and rudder control, δ_{ad} , δ_{rad} :

$$\begin{bmatrix} \delta_{ad}(t) \\ \delta_{rad}(t) \end{bmatrix} = \begin{bmatrix} \hat{\theta}_1^T(t) \Phi_{1,1}(\beta(t), p_s(t), r_s(t)) + \hat{w}_1^T(t) \Phi_{1,2}(\beta(t), p_s(t), r_s(t), \delta_{a_{nom}}(t) - \delta_{ad}(t)) \\ \hat{\theta}_2^T(t) \Phi_{2,1}(\beta(t), p_s(t), r_s(t)) + \hat{w}_2^T(t) \Phi_{2,2}(\beta(t), p_s(t), r_s(t), \delta_{r_{nom}}(t) - \delta_{rad}(t)) \end{bmatrix}$$

where $\delta_{a_{nom}}$ and $\delta_{r_{nom}}$ have been defined in Eq. (39). The fast dynamics are designed as

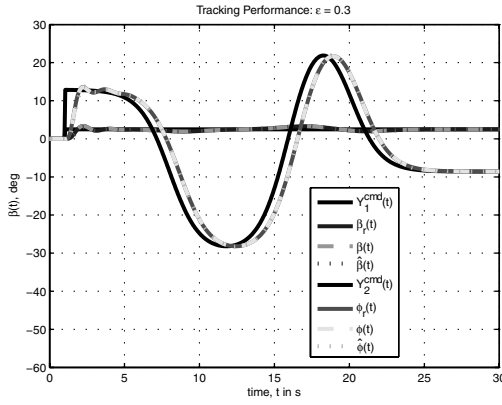
$$-0.3 \begin{bmatrix} \dot{\delta}_{ad}(t) \\ \dot{\delta}_{rad}(t) \end{bmatrix} = \bar{\mathbf{P}}(t, e, \delta_{ad}(t), \delta_{rad}(t)) \mathbf{f}(t, e, \delta_{ad}(t), \delta_{rad}(t))$$

where

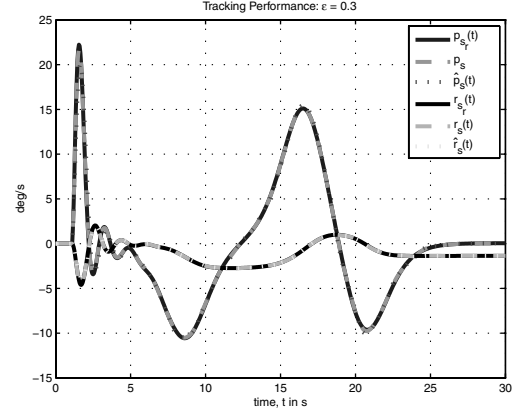
$$\bar{\mathbf{P}}(t, e, \delta_{ad}, \delta_{rad}) = \text{diag} \left\{ \begin{array}{l} -\hat{w}_1^T(t) \frac{\partial \Phi_{1,2}}{\partial \delta_{ad}}(\beta(t), p_s(t), r_s(t), \delta_{a_{nom}}(t) - \delta_{ad}(t)) + 1 \\ -\hat{w}_2^T(t) \frac{\partial \Phi_{2,2}}{\partial \delta_{rad}}(\beta(t), p_s(t), r_s(t), \delta_{r_{nom}}(t) - \delta_{rad}(t)) + 1 \end{array} \right\}$$

and

$$\mathbf{f}(t, e, \delta_{ad}, \delta_{rad}) = \begin{bmatrix} \delta_{ad}(t) - \hat{\theta}_1^T(t) \Phi_{1,1}(\beta(t), p_s(t), r_s(t)) - \hat{w}_1^T(t) \Phi_{1,2}(\beta(t), p_s(t), r_s(t), \delta_{a_{nom}}(t) - \delta_{ad}(t)) \\ \delta_{rad}(t) - \hat{\theta}_2^T(t) \Phi_{2,1}(\beta(t), p_s(t), r_s(t)) - \hat{w}_2^T(t) \Phi_{2,2}(\beta(t), p_s(t), r_s(t), \delta_{r_{nom}}(t) - \delta_{rad}(t)) \end{bmatrix}$$

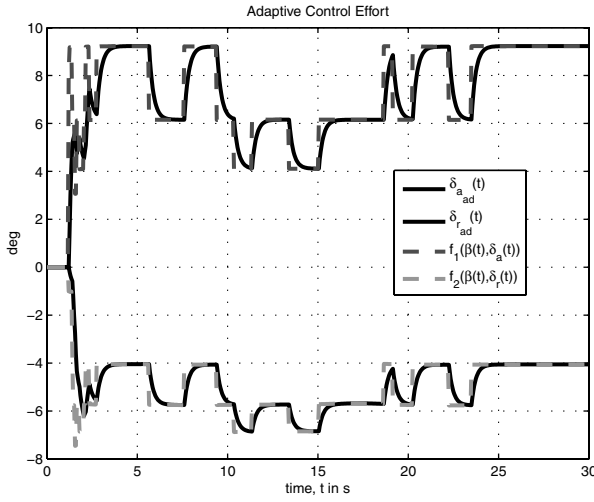


a) States $\hat{\beta}(t), \beta_r(t), \beta(t), \hat{\phi}(t), \phi_r(t), \phi(t)$ of roll/yaw system

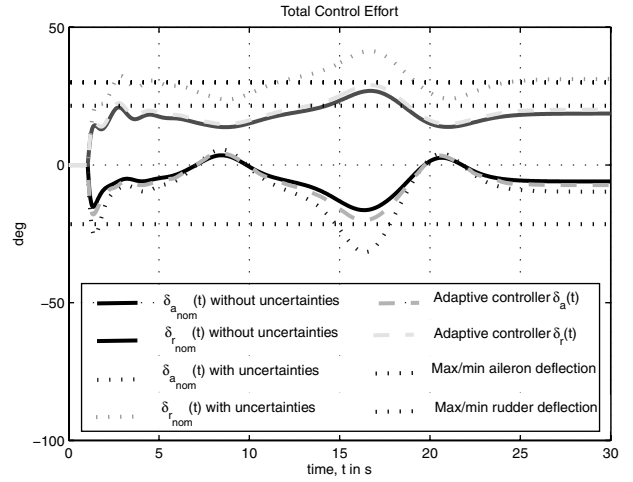


b) States $\hat{p}_s(t), p_{s_r}(t), p_s(t), \hat{r}_s(t), r_{s_r}(t), r_s(t)$ of roll/yaw system

Fig. 11 System performance.



a) Adaptive control effort of roll/yaw system compared to uncertainties f_1 and f_2



b) Control efforts: PI with no uncertainties and adaptive with uncertainties are within the control limits, while baseline PI with uncertainties is outside the aileron and rudder actuator limits

Fig. 12 System performance with adaptation.

The plots in Figs. 11 and 12 show the tracking performance with the augmented adaptive controller. Asymptotic tracking of angles β and ϕ is achieved while the other states of the system p_s and r_s remain bounded. Additionally, the approximations stay within their respective domains. Figure 12a shows the time history of the adaptive control signal and the unknown nonlinearities $f_1(\beta(t), p_s(t), r_s(t), \delta_a(t))$ and $f_2(\beta(t), p_s(t), r_s(t), \delta_r(t))$ as functions of time. Again, Fig. 12 shows that using adaptation reduces the control effort to within reasonable ranges as compared to the PI controller.

IV. Conclusion

In this paper, an approximate dynamic inversion based adaptive control design methodology is presented for the nonlinear-in-control uncertain short-period and lateral/directional aircraft dynamics via time-scale separation. System nonlinearities are approximated via two sets of basis functions, one of which retains the sign of the system's control effectiveness, while the other set is control independent. The adaptive controller is further introduced as a solution of fast dynamics, which achieves time-scale separation between the system dynamics and the controller dynamics. Simulations of the short-period and roll/yaw systems have verified the benefits of this method.

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